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ADDENDUM

Addendum to 'Completeness of the Bethe Ansatz solution of the open XXZ chain with nondiagonal boundary terms'

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In [1] (to which we refer to hereafter by I), we find significant numerical evidence that for the open XXZ quantum spin chain Hamiltonian (I1.1) with bulk and boundary parameters satisfying the constraint (I1.2)

$$\alpha_- + \beta_- + \alpha_+ + \beta_+ = \pm(\theta_- - \theta_+) + \eta k \tag{1}$$

the Bethe Ansatz solution (I1.3)–(I1.8), (I2.9) gives the complete set of 2^N eigenvalues for $k = N + 1$; and this solution gives only some (but not all) of the eigenvalues for $1 - N \leqslant k < N + 1$. Here we conjecture that a simple generalization of this Bethe Ansatz solution gives the *complete* set of eigenvalues for *all* values of k in the interval $-(N + 1) \leqslant k \leqslant N + 1$.

Indeed, consider the following two expressions, distinguished by $\pm$, for the eigenvalues of the transfer matrix (I2.4):

$$\Lambda_\pm(u) = h_\pm(u)\frac{Q_\pm(u-\eta)}{Q_\pm(u)} + h_\pm(-u-\eta)\frac{Q_\pm(u+\eta)}{Q_\pm(u)} \tag{2}$$

where $h_\pm(u)$ are given by

$$h_\pm(u) = -\sinh^{2N}(u+\eta)\frac{\sinh(2u+2\eta)}{\sinh(2u+\eta)} \times 4\sinh(u\pm\alpha_-)\cosh(u\pm\beta_-)\sinh(u\pm\alpha_+)\cosh(u\pm\beta_+) \tag{3}$$

and $Q_\pm(u)$ are given by

$$Q_\pm(u) = \prod_{j=1}^{M_\pm}\sinh\left(u - u_j^\pm\right)\sinh\left(u + u_j^\pm + \eta\right) \tag{4}$$

where

$$M_\pm = \tfrac{1}{2}(N - 1 \pm k). \tag{5}$$

The corresponding Bethe Ansatz equations are

$$\frac{h_{\pm}\left(u_j^{\pm}\right)}{h_{\pm}\left(-u_j^{\pm}-\eta\right)} = -\frac{Q_{\pm}\left(u_j^{\pm}+\eta\right)}{Q_{\pm}\left(u_j^{\pm}-\eta\right)} \qquad j = 1, \ldots, M_{\pm}. \tag{6}$$

The Bethe Ansatz solution considered earlier [1, 2] corresponds to the above expressions with the plus (+) sign.

We conjecture that for a given set of bulk and boundary parameters satisfying (1) with $|k| \leqslant N + 1$ and k odd (even) for N even (odd), the eigenvalues $\Lambda_{+}(u)$ and $\Lambda_{-}(u)$ together give the complete set of eigenvalues of the transfer matrix. That is, for $k = N + 1$, all the eigenvalues are given by $\Lambda_{+}(u)$ (as found in I); for $k = -(N+1)$, all the eigenvalues are given by $\Lambda_{-}(u)$; and for $-(N + 1) < k < N + 1$, both $\Lambda_{+}(u)$ and $\Lambda_{-}(u)$ are needed to obtain the complete set of eigenvalues.

This conjecture is supported by significant numerical evidence, obtained (as in I) using 'McCoy's method'. For instance, for $N = 6$ and $k = 1$, we find that 42 eigenvalues are given by $\Lambda_{+}(u)$ and 22 eigenvalues are given by $\Lambda_{-}(u)$; together, they give the complete set of $2^6 = 64$ eigenvalues.

References

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